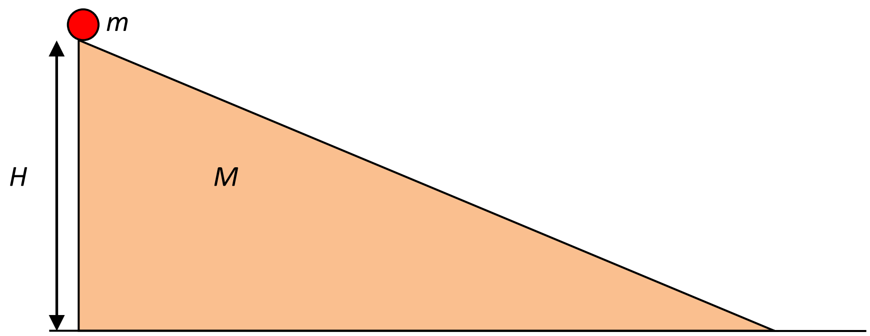
Teacher notes
Topic A

Ball on movable inclined plane

A ball of mass *m* is released from rest from the top of an inclined plane.



Determine the speed of the ball when it reaches level ground when (i) the inclined plane cannot move and (ii) when it can move without friction on level ground. The mass of the inclined plane is *M*.

(i) The first part is straightforward: by energy conservation, $mgH = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gH}$.

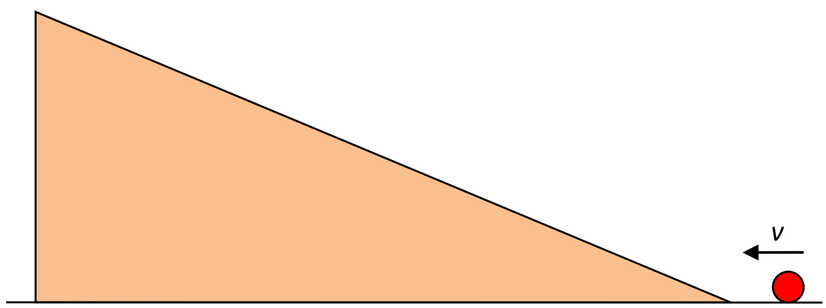
(ii) For the second part: conservation of momentum gives $0 = mv - Mu$ where *u* is the speed of the inclined plane to the left and *v* that of the ball to the right.

Conservation of energy gives: $mgH = \frac{1}{2}mv^2 + \frac{1}{2}Mu^2$. Substituting $u = \frac{m}{M}v$, we get

$$mgH = \frac{1}{2}mv^2 + \frac{1}{2}M(\frac{m}{M}v)^2$$
$$2mgH = mv^2 + \frac{m^2v^2}{M}$$
$$2gH = v^2 + \frac{mv^2}{M} = v^2(1 + \frac{m}{M})$$
$$v = \sqrt{\frac{2gH}{1 + \frac{m}{M}}}$$

In the limit $M \to \infty$, i.e., when the inclined plane mass is very much greater than the mass of the ball, we have that $v = \sqrt{\dfrac{2gH}{1+\dfrac{m}{M}}} \to \sqrt{2gH}$, the same result as the case of the immovable inclined plane.

As a variant, we now consider the ball to be moving at speed v on level ground and climbing on the inclined plane. How high will it get?



At the highest point the ball and the inclined plane will have (for an instant) a common speed *u*. By conservation of momentum $mv = (m+M)u$.

By energy conservation $\frac{1}{2}mv^2 = mgh + \frac{1}{2}(M+m)u^2$. Hence

$$\frac{1}{2}mv^2 = mgh + \frac{1}{2}(M+m)\left(\frac{mv}{M+m}\right)^2$$
$$\frac{1}{2}mv^2 = mgh + \frac{1}{2}\frac{m^2v^2}{M+m}$$
$$v^2 = 2gh + \frac{mv^2}{M+m}$$
$$2gh = (1-\frac{m}{M+m})v^2$$
$$h = (1-\frac{m}{M+m})\frac{v^2}{2g}$$
$$h = \frac{M}{M+m}\frac{v^2}{2g}$$

In the limit $M \to \infty$, we have that $h \to \frac{v^2}{2g}$, the same result as the case of the immovable inclined plane.